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A new proposal for Lagrangian correlation coefficient ☆

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Abstract

The statistical description of dispersion in turbulent flow was first considered by Taylor (Proc. London Math. Soc. 20 (1921) 196) and the statistical properties of the field were determined by Lagrangian correlation coefficient $R_{\mathrm{L}}(\tau)$. Frenkiel (Adv. Appl. Mech. 3 (1953) 61) has proposed several simple forms for $R_{\mathrm{L}}(\tau)$. Some workers have investigated for a proper form of the Lagrangian correlation coefficient. In this work, a new proposal for the Lagrangian correlation coefficient is proposed and discussed. It can be written in general form with the one of the Frenkiel's (Adv. Appl. Mech. 3 (1953) 61) Lagrangian correlation coefficient. There is very satisfactory agreement between the new correlation and the experiment.

Keywords: Turbulent flow; Mean squared displacement; Lagrangian correlation coefficient

1. Introduction

In discussing the dispersion of fluid in a turbulent flow, Taylor's (1921) turbulent diffusion theory is often quoted and many theoretical and experimental aspects of this have been investigated. When studying dispersion of fluid particles, Lagrangian statistical characteristics such as the Lagrangian velocity auto-correlation are important concepts. The Lagrangian correlation coefficient $R_{\mathrm{L}}(\tau)$ has asymptotic forms in the limiting cases of very short and very long dispersion times. However, the dispersion is expected at intermediate times to depend on the manner in which the $R_{\mathrm{L}}(\tau)$ decays with time and there is little information on this point. After Frenkiel (1953) had proposed several basic forms for $R_{\mathrm{L}}(\tau)$ in addition to the exponential form, several researchers have used it in the modelling and analysis of their data (Berlemont et al., 1990; Taylor, 1974).

Lagrangian statistics are extremely difficult to obtain experimentally because they require measurements following the random motion of fluid particles (Snyder and Lumley, 1971; Taylor and Middleman, 1974; Shlien and Corrsin, 1974; Sato and Yamamoto, 1987; Call and Kennedy, 1991; Altınsoy, 1997; Altınsoy and Tuğrul, 1999; Sato et al., 2000). The objective of the work reported here was to investigate empirical model functions of the Lagrangian correlation coefficients. A new form of the Lagrangian correlation coefficient was proposed, and its reliability was discussed.

2. Theory

The fundamental equation relates the mean squared displacement $\overline{y^2}$ to the mean squared fluctuating velocity $\overline{u^2}$ in the direction transverse to the mean flow, and to the Lagrangian correlation coefficient $R_{\mathrm{L}}(\tau)$ (Taylor, 1921):

$$\overline{y^2} = 2\overline{u^2}\int_0^t \int_0^{t'} R_{\mathrm{L}}(\tau)\,\mathrm{d}\tau\,\mathrm{d}t' \tag{1}$$

The Lagrangian velocity correlation coefficient is defined as:

$$R_{\mathrm{L}}(\tau) = \frac{\overline{u(t)u(t-\tau)}}{\overline{u(t)}^2} \tag{2}$$

Frenkiel (1953) has proposed several basic forms for $R_{\mathrm{L}}(\tau)$ which have semi-empirical bases:

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Nomenclature

m	loop parameter
$R_L(\tau)$	Lagrangian correlation coefficient
$\overline{u^2}$	mean squared fluctuating velocity
t	time
T_L	Lagrangian integral time scale
$\overline{y^2}$	mean squared displacement
Greek	
τ	time interval

$$R_L(\tau) = \exp\left[-\frac{\tau}{T_L}\right] \tag{3}$$

$$R_L(\tau) = \exp\left[-\frac{\tau}{2T_L}\right]\cos\left[\frac{\tau}{2T_L}\right] \tag{4}$$

$$R_L(\tau) = \exp\left[-\frac{\pi\tau^2}{4T_L^2}\right] \tag{5}$$

The mean square displacements were found by substituting Eqs. (3)–(5) into Eq. (1) to give, respectively:

$$\overline{y^2} = 2\overline{u^2}T_L[t - T_L[1 - \exp(-t/T_L)]] \tag{6}$$

$$\overline{y^2} = 2\overline{u^2}T_L[t - 2T_L\exp(-t/2T_L)\sin(t/2T_L)] \tag{7}$$

$$\overline{y^2} = 2\overline{u^2}T_L\left[-t\,\mathrm{erf}\left(\frac{\sqrt{\pi}}{2T_L}t\right) + \frac{2T_L}{\pi}\exp\left(-\frac{\pi}{4T_L^2}t^2\right) - \frac{2T_L}{\pi}\right] \tag{8}$$

We propose a new correlation:

$$R_L(\tau) = \exp\left[-\frac{\pi\tau^2}{8T_L^2}\right]\cos\left[\frac{\tau^2}{2T_L^2}\right] \tag{9}$$

New correlation in Eq. (9) is being examined with the requirements of Hinze (1975). It meets all of them.

Known Lagrangian correlation coefficients (Eqs. (3)–(5)) with the new correlation (Eq. (9)) could be bring into view two main Lagrangian correlation coefficient sets. The general formula for the first main Lagrangian correlation coefficient set was expressed by Frenkiel (1953):

$$R_L(\tau) = \exp\left[-\frac{\tau}{(m^2+1)T_L}\right]\cos\left[\frac{m\tau}{(m^2+1)T_L}\right] \tag{10}$$

Eq. (10) can be turned into Eq. (3) for $m = 0$ and can be turned into Eq. (4) for $m = 1$. Then the general formula of the new proposal for the second main Lagrangian correlation coefficient set can be formed as:

$$R_L(\tau) = \exp\left[-\frac{\pi\tau^2}{4(m^2+1)T_L^2}\right]\cos\left[\frac{m\tau^2}{(m^2+1)T_L^2}\right] \tag{11}$$

Now, this form of the Lagrangian correlation coefficient in Eq. (11), can be turned into Eq. (5) for $m = 0$ and can be turned into the new correlation coefficient in Eq. (9) for $m = 1$.

3. Results and discussion

Turbulent pipe flow was examined by measuring the radial dispersion of dyed fluid introduced at an axial point source within the flow by Taylor and Middleman (1974). Results of the calculation of the mean squared displacement $\overline{y^2}$ vs. the diffusion time t for data of Taylor and Middleman (1974) are given for 45,000 Reynolds number by using two main Lagrangian correlation coefficient sets in Figs. 1 and 2.

$m = 0$ and $m = 1$ are more appropriate each other for two different main Lagrangian correlation coefficient sets and they are preferable forms for the practical using (Berlemont et al., 1990; Graham and James, 1996; Groenhof, 1970; Hanratty et al., 1956; Kalinske and Pien, 1944; Taylor, 1974). So, Eq. (9) may be proposed as a new Lagrangian correlation coefficient.

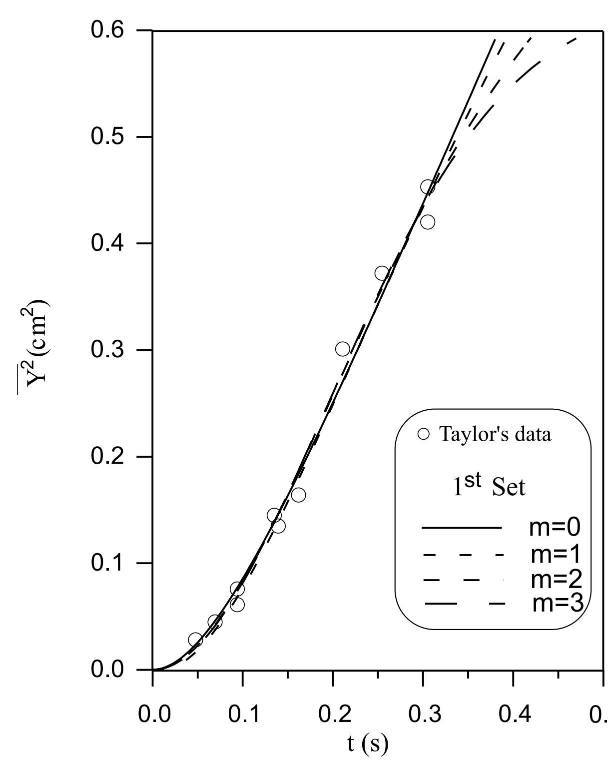

Fig. 1. Mean squared displacement vs. time. Comparison of first Lagrangian correlation coefficient set by using Taylor and Middleman (1974) experimental data for $Re = 45{,}000$.

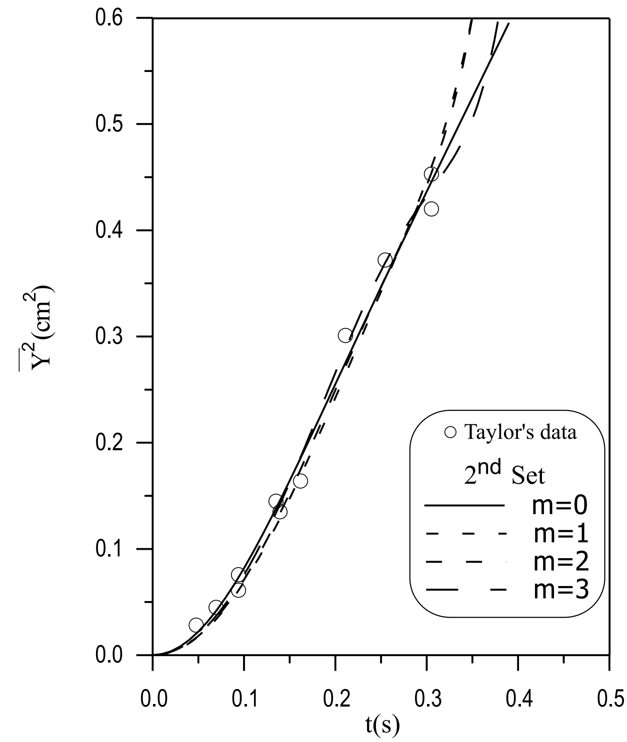

Fig. 2. Mean squared displacement vs. time. Comparison of second Lagrangian correlation coefficient set by using Taylor and Middleman (1974) experimental data for $Re = 45{,}000$.

With the proposal of new Lagrangian correlation coefficient that supply the requirements for the Lagrangian correlation coefficient was shown agreement with the dye tracer experimental data (Taylor and Middleman, 1974).

It can be concluded that:

1. A new Lagrangian correlation coefficient is proposed in Eq. (9). This correlation coefficient meets all the requirements defined by Hinze (1975). So, the new Lagrangian correlation coefficient can be proposed as a Lagrangian correlation coefficient in addition to Frenkiel (1953) correlation.
2. All the Lagrangian correlation coefficients can be grouped in two main sets (Eqs. (10) and (11)).
3. The new model is in acceptable agreement with the experimental data.
4. For the two main Lagrangian correlation coefficient sets, it can be noticed that $m = 0$ and $m = 1$ are more preferable for the experimental data.

References

Altınsoy, N., 1997. Investigation of turbulent flow in the vertical pipes and a new correlation, Ph.D. Thesis, Institute For Nuclear Energy, Istanbul Technical University, Istanbul.

Altınsoy, N., Tuğrul, A.B., 1999. A radiotracer application for the turbulent dispersion of fluids. Appl. Radiat. Isot. 51 (4), 367–375.

Berlemont, A., Desjonqueres, P., Gousbet, G., 1990. Particle lagrangian simulation in turbulent flow. Int. J. Multiphase Flow 16, 19–34.

Call, C.J., Kennedy, I.M., 1991. A technique for measuring Lagrangian and Eularian particle statistics in a turbulent flow. Exp. Fluids 12, 125–130.

Frenkiel, F.N., 1953. Turbulent diffusion: Mean concentration distribution in a flow field of homogeneous turbulence. In: Mises, R.V., Karman, T.V. (Eds.), Adv. Appl. Mech., 3. Academic Press, New York, 61–107.

Graham, D.I., James, P.W., 1996. Turbulent dispersion of particles using eddy interaction models. Int. J. Multiphase Flow 22 (1), 157–175.

Groenhof, H.C., 1970. Eddy diffusion in the central region of turbulent flows in pipes and between parallel plates. Chem. Eng. Sci. 25, 1005–1014.

Hanratty, T.J., Latinen, G., Wilhelm, R.H., 1956. Turbulent diffusion in particulately fludized beds of particles. AIChE 2, 372–380.

Hinze, J.O., 1975. Turbulence. McGraw Hill Company, New York.

Kalinske, A.A., Pien, C.L., 1944. Eddy diffusion. Ind. Eng. Chem. 36, 220–223.

Sato, Y., Fukuichi, U., Hishida, K., 2000. Effect of inter-particle on turbulence modulation by Lagrangian PIV. Int. J. Heat Fluid Flow 21 (5), 554–561.

Sato, Y., Yamamoto, K., 1987. Lagrangian measurement of fluid-particle motion in an isotropic turbulent field. J. Fluid Mech. 175, 183–199.

Shlien, D.J., Corrsin, S., 1974. A measurement of Lagrangian velocity autocorrelation in approximately isotropic turbulence. J. Fluid Mech. 62 (2), 255–270.

Snyder, W.H., Lumley, J.L., 1971. Some measurements of particle velocity autocorrelation functions in a turbulent flow. J. Fluid Mech. 48, 41.

Taylor, A.R., 1974. Turbulent dispersion in drag reducing fluids, Ph.D. Thesis, University of Massachusetts, UMI Dissertation Services.

Taylor, A.R., Middleman, S., 1974. Turbulent dispersion in drag-reducing fluids. AIChE 20 (3), 454–461.

Taylor, G.I., 1921. Diffusion by continuous movements. Proc. London Math. Soc. 20, 196.